Class 12 TNEB Physics

RAY OPTICS

1. Relation between focal length and radius of curvature:

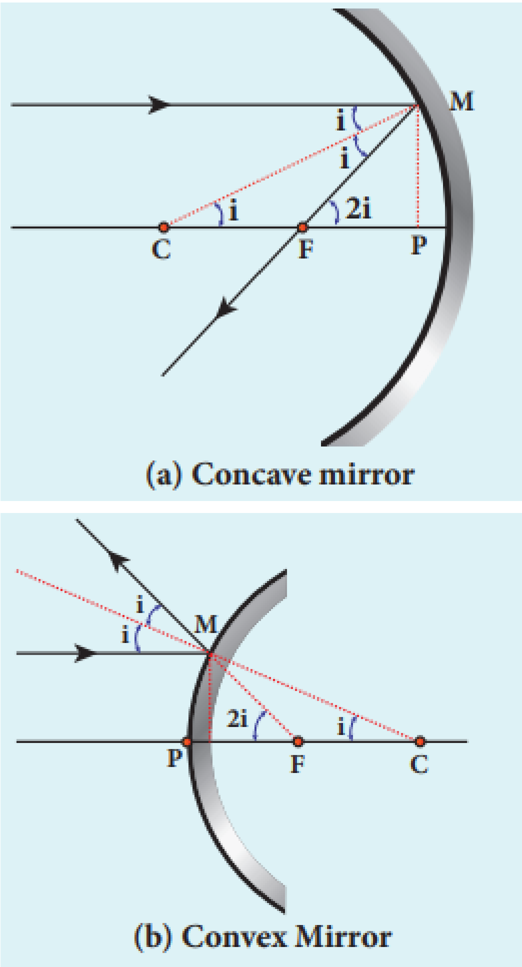

Fig.-1: relation between R & f

If MP is the perpendicular from M on the principal axis, then from the geometry,

The angles ∠MCP = i and ∠MFP = 2i

From right angle triangles ΔMCP and ΔMFP,

$\tan \boldsymbol{i} = PM/PC$ and $\tan \boldsymbol{2i} = PM/PF$-----(1)

As the angles '*i*' and '*2i*' are small, *tan i =i* and *tan2i =2i*

Hence $i = \frac{PM}{PC}$ and $2i = \frac{PM}{PF}$------(2)

Simplifying further, from eqn. (1) & (2)

$2\frac{PM}{PC} = \frac{PM}{PF}$

cancelling PM on both sides yields- $\frac{2}{PC} = \frac{1}{PF}$

Hence *2PF= PC*

PF is *focal length* ***f*** and *PC* is the *radius of curvature* ***R***

i.e. ***2f=R (or) f=R/2***

2. Mirror equation

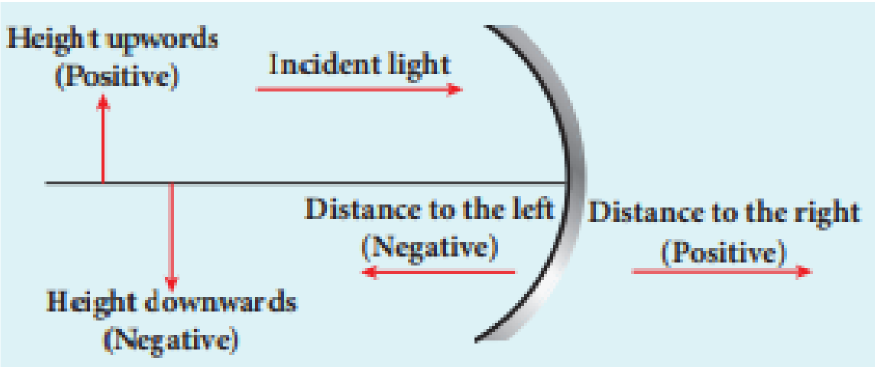

Fig.-2: Cartesian sign convention

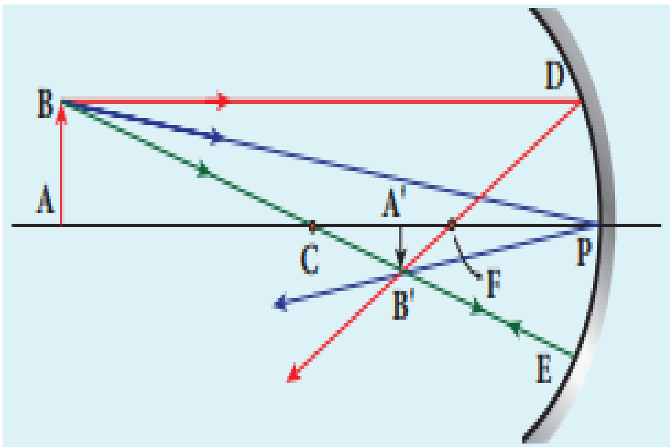

Fig.-3: Mirror Equation

As per law of reflection, the angle of incidence ∠BPA is equal to the angle of reflection ∠ B′PA′. The triangles ΔBPA and ΔB′PA′ are similar. Thus, from the rule of similar triangles,

$$\frac{A'B'}{AB} = \frac{PA}{PA'} \text{ ----------------(1)}$$

The other set of similar triangles are, ΔDPF and ΔB′A′ F

PD is almost a straight vertical line, (assuming large curvature of Mirror and D very close to P)

$$\frac{A'B'}{PD} = \frac{A'F}{PF}$$

As, the distances PD = AB the above equation becomes,

$$\frac{A'B'}{AB} = \frac{A'F}{PF} \text{-----------(2)}'$$

From equations (1) and (2) we can write,

$$\frac{PA'}{PA} = \frac{A'F}{PF}$$

As A′ F = PA' - PF , the above equation becomes,

$$\frac{PA'}{PA} = \frac{PA' - PF}{PF} \text{----------(3)}$$

We can apply the sign conventions for the various distances in the above equation.

PA = $-u$ $PA' = -v$ PF = $-f$

All the three distances are negative as per sign convention, because they are measured to the left of the pole. Now, the equation (3) becomes

$$\frac{-v}{-u} = \frac{-v - (-f)}{-f}$$

On further simplification

$$\frac{v}{u} = \frac{-(v + f)}{-f}$$

$$\text{i.e. } \frac{v}{u} = \frac{v - f}{f}$$

$$\frac{v}{u} = \frac{v}{f} - 1$$

Dividing either side with v,

$$\frac{1}{u} = \frac{1}{f} - 1/v$$

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

After rearranging,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{----------}(4)$$

The above equation (4) is called **mirror equation**. Although this equation is derived for a special situation shown in Figure (3), it is also valid for all other situations *with any spherical mirror*. This is because proper sign convention is followed for *u*, *v* and *f* in equation (3).

3. Fizeau's method for measuring Speed of Light

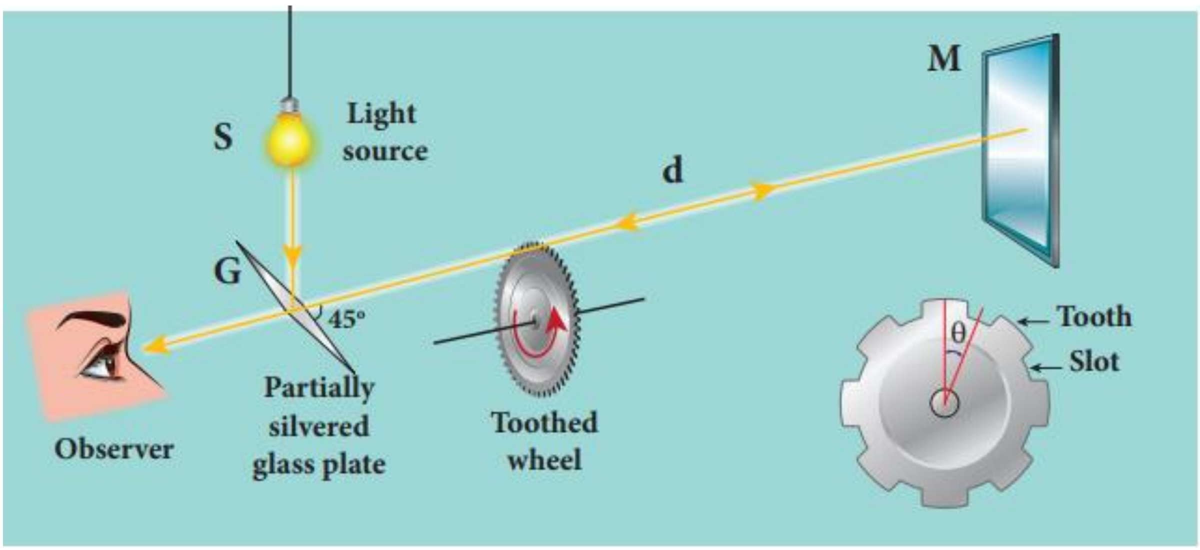

Fig-4 Speed of light by Fizeau's method

Description of the Measuring Equipment

The light from the source S was first allowed to fall on a partially silvered glass plate G kept at an angle of 45° to the incident light from the source. The light then was allowed to pass through a rotating toothed wheel with N teeth and N cuts of equal widths whose speed of rotation could be varied through an external mechanism (not shown in the Figure).

The light passing through one cut in the wheel will get reflected by a mirror M kept at a long distance d, about 8 km from the toothed wheel. If the toothed wheel was not rotating, the reflected light from the mirror would again pass through the same cut and reach the eyes of the observer through the partially silvered glass plate

Working:

The angular speed of rotation of the toothed wheel was increased from zero to a value ω until light passing through one cut would completely be blocked by the adjacent tooth. This is ensured by the disappearance of light while looking through the partially silvered glass plate.

Expression for speed of light:

The speed of light in air '$\boldsymbol{v}$' is equal to the ratio of the distance the light travelled from the toothed wheel to the mirror and back '$\boldsymbol{2d}$' to the time taken '$\boldsymbol{t}$'

$$v = \frac{\boldsymbol{2d}}{\boldsymbol{t}} \text{-----------}(5)$$

The distance '$\boldsymbol{d}$' is a known value from the arrangement. The time taken t for the light to travel the distance to and fro, is calculated from the angular speed '$\boldsymbol{\omega}$' of the toothed wheel.

$$\boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\boldsymbol{t}} \text{------------}(6)$$

Here, '$\boldsymbol{\theta}$'is the angle between the tooth and the slot which is rotated by the toothed wheel within that time '$\boldsymbol{t}$'

$$\boldsymbol{\theta} = \frac{\text{total angle of the circle in Radians}}{\text{number of teeth} + \text{number of cuts}}$$

i.e. $\boldsymbol{\theta} = \frac{\boldsymbol{2\pi}}{\boldsymbol{2N}} = \frac{\boldsymbol{\pi}}{\boldsymbol{N}}$

Substituting for θ in the equation (6) for $\boldsymbol{\omega}$

$$\boldsymbol{\omega} = \frac{\pi/N}{t} = = \frac{\pi}{\boldsymbol{Nt}}$$

Rewriting the above equation for ***'t'***

$$\boldsymbol{t} = \frac{\pi}{\boldsymbol{N\omega}}\text{----------(7)}$$

Substituting t from equation (6.14) in equation (5),

$$v = \frac{\boldsymbol{2d}}{\boldsymbol{t}} = \frac{\boldsymbol{2d}}{\pi/N\omega}$$

After rearranging,

$$v = \frac{\boldsymbol{2dN\omega}}{\boldsymbol{\pi}}\text{--------------}\mathbf{(8)}$$

Refractive index

Refractive index of a transparent medium is defined as the ratio of speed of light in vacuum (or air) to the speed of light in that medium.

refractive index ***n*** of a medium $= \frac{\text{speed of light in Vacuum}(\boldsymbol{c})}{\text{speed of light in medium}(\boldsymbol{v})}$

$$n = \frac{c}{v}\text{--------------(9)}$$

Note:

- Refractive index does not have unit.
- The smallest value of refractive index is for vacuum, which is 1.
- For any *other medium* refractive index is >1.
- Refractive index is also called as ***optical density of the medium***.
- *Higher* the *refractive index* of a medium, greater is ***its optical density*** and *speed of light through the medium is lesser and vice versa.*

Optical path

Optical path of a medium is defined as the distance 'd' light travels in vacuum in the same time it travels a distance d' in the medium

Let us consider a medium of refractive index ***n*** and thickness ***d***. Light travels with a speed ***v*** through the medium in a time ***t***.

Then we can write,

$v = \frac{d}{t}$ rewritten as , $t = \frac{d}{v}$--------(a)

In the same time, light can cover a greater distance ***'d'*** in vacuum as it travels with greater speed '***c***' in vacuum

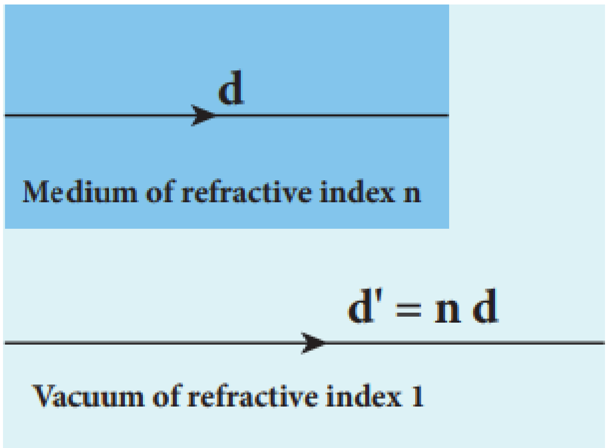

Fig. 7-Optical path

$c = \frac{d'}{t}$ rewritten as, $t = \frac{d'}{c}$--------(b)Type equation here.

from equations (a) and (b),

$\frac{d'}{c} = \frac{d}{v}$

$d' = \frac{c}{v} d$---------------(c)

as $\frac{c}{v} = n$, as per eqn (9)

∴ equation (c) can be rewritten as;

∴ $d' = nd$ (The optical path is d'[as seen from Fig.(7)]

As ***n*** is always > 1, the optical path ***d'*** of the medium is always less than ***d***.

4. **Refraction**

Law of refraction is called Snell's law. Snell's law states that,

a) The incident ray, refracted ray and normal to the refracting surface are all coplanar (ie. lie in the same plane).

b) The ratio of angle of incidence '***i*** 'in the first medium to the angle of reflection '***r'*** in the second medium is equal to the ratio of refractive index of the second medium ***n2*** to that of the refractive index of the first medium n1 .

$$\frac{\sin i}{\sin r} = \frac{n2}{n1} \quad \text{-------------}(9)$$

The above equation is in the ratio form. It can also be written in a much useful product form as,

n1sini=n2sinr---------------(10)

The refraction at a boundary is shown in Figure 5

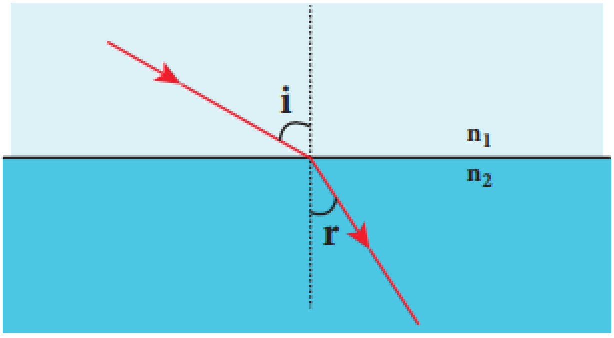

Fig.:5-Refraction of Light

Angle of Deviation due to refraction

Angle between the incident ray and refracted light ray is called the

Angle of deviation(d)

When light travels from *rarer to denser medium* it <u>deviates towards normal</u> as shown in Figure 6.

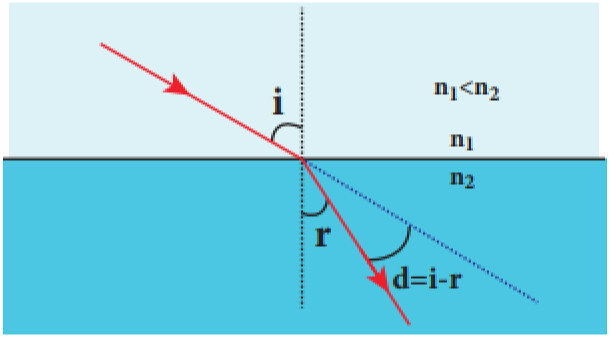

Figure 6.: *Angle of deviation due to refraction from rarer to denser medium*

In this case, the Angle of Deviation-(d) is given by the relation

$$d = i - r \text{-----------------------}(10)$$

On the other hand, if light travels from denser to rarer medium it deviates ***away*** from normal as shown in Figure 7. The angle of deviation in this case is given by,

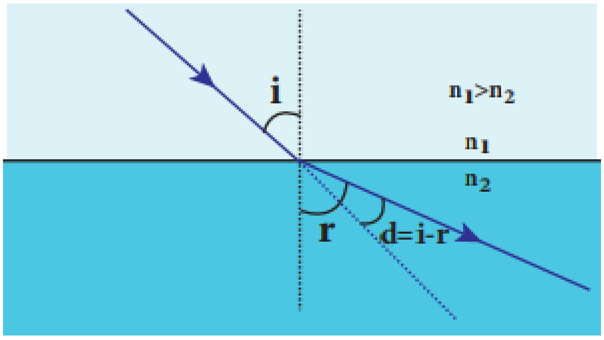

Fig. (7)- Angle of deviation due to refraction from denser to rarer medium

Characteristics of refraction

a) When light passes from rarer medium to denser medium it deviates towards normal in the denser medium.

b) When light passes from denser medium to rarer medium it deviates away from normal in the rarer medium.

c) ***In any refracting surface there will also be some reflection taking place.*** Thus, the intensity of refracted light will be lesser than the incident light.

The phenomenon in ***which a part of light from a source undergoing reflection and the other part of light from the same source undergoing refraction at the same surface is called*** ***<u>simultaneous reflection and refraction.</u>***

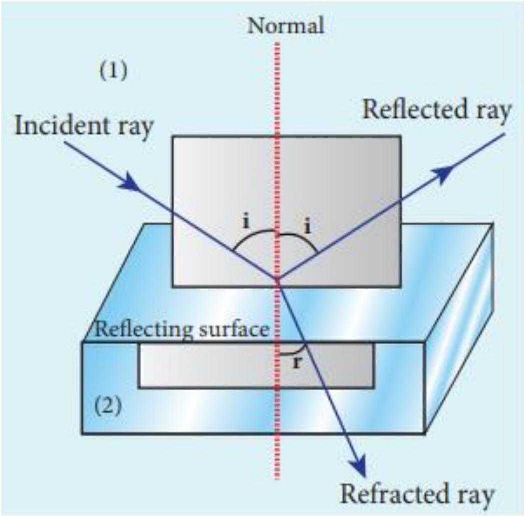

Figure 8- Simultaneous reflection and refraction

Principle of reversibility

The principle of reversibility states that light will follow exactly the same path if its direction of travel is reversed. This is true for both reflection and refraction as shown in Figure 9

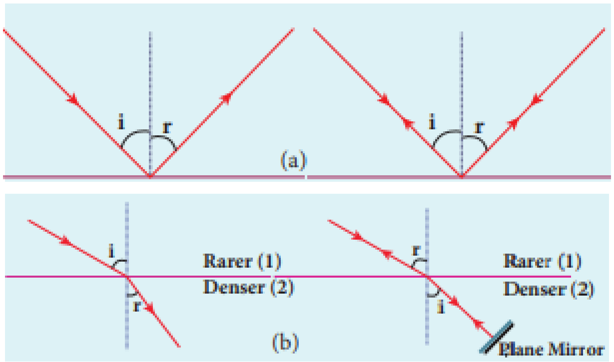

Figure 6.19 Principle of reversibility in (a) reflection and (b) refraction

Relative Refractive Index

In the equation for Snell's law, the term $\frac{n1}{n2}$ is called relative refractive index of *second medium*(2) with respect to the *first medium(1),* which is denoted as (n_{21}).

$$\boldsymbol{n21 = \frac{n2}{n1}}$$

The concept of relative refractive index gives rise to other useful relation such as,

a) Inverse rule:

$$\boldsymbol{n12 = \frac{1}{n21} \quad or \quad \frac{n1}{n2} = \frac{1}{\frac{n1}{n2}}}$$

5. Lens formula and lens makers formula

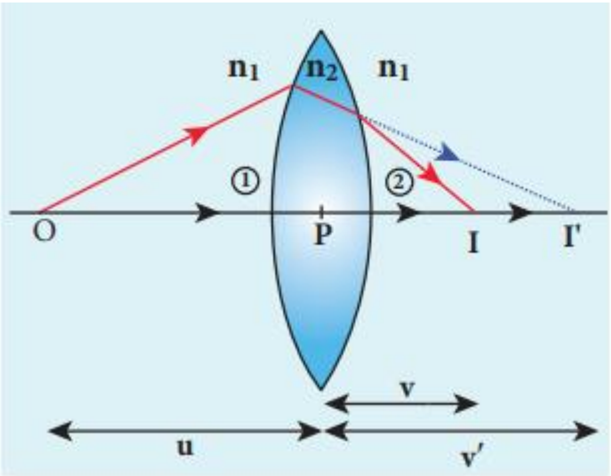

Fig.: 7-Refraction through thin lens

Let us consider a thin lens made up of a medium of refractive index n2 is placed in a medium of refractive index n1 . Let R1 and R2 be the radii of curvature of two spherical surfaces ① and ② respectively and P be the pole as shown in figure 7.

Consider a point object O on the principal axis. The ray which falls very close to P, after refraction at the surface ① forms image at I'. Before it does so, it is again refracted by the surface ②. Therefore the final image is formed at I.

The general equation for the refraction at a spherical surface is given from Equation (6.59),

$$\frac{n2}{v} - \frac{n1}{u} = \frac{(n2 - n1)}{R}$$